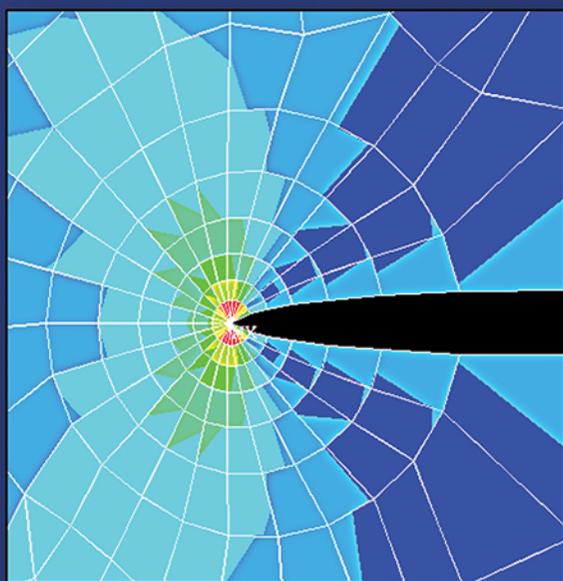


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# APPLICATION OF FINITE ELEMENT ANALYSIS FOR FRACTURE AND DAMAGE MECHANICS



ZHAOCHUN YANG



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# Application of Finite Element Analysis for Fracture and Damage Mechanics

*Application of Finite Element Analysis for Fracture and Damage Mechanics* focuses on the finite element analysis of various material models and their relevant fracture and damage models using this advanced software. The book consists of three parts: Part I introduces the various material models, Part II highlights the finite element modeling of their relevant fracture parameters, and Part III deals with various material damage models. Each part begins with a simple problem with theoretical results compared with finite element results to help readers understand the numerical simulation results.

- Discusses material models such as composite, elastic, elastoplastic, and creep models
- Covers fracture parameters like  $K$ ,  $J$ -Integral,  $C^*$ -Integral, and VCCT
- Presents relevant material damage models (ductile, creep, and composite)
- Explores typical and complex problems, including the inclined crack model, penny shaped crack model, compact tension specimen, end-loaded split (ELS) model, double cantilever beam (DCB) model, and notched model
- Includes all modeling files, such as APDL input files, Python code, and creep damage subroutine, in the appendixes

Written for materials and mechanical engineers, this text addresses and provides solutions to the real-life engineering challenge of damage and fracture in materials and structures.

**Zhaochun Yang** received his PhD from the University of Pittsburgh in 2004. He has been an FEA Engineer since 2007. Dr. Yang has published three books by CRC Press: *Finite Element Analysis for Biomedical Engineering Applications*; *Multiphysics Modeling with Application to Biomedical Engineering*; and *Material Modeling in Finite Element Analysis*.



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# Preface

After I earned my PhD from the University of Pittsburgh, I joined the research & development department of a large company in 2007. I have been working with material and fracture modeling for approximately twenty years and have gained much experience in the finite element field. I have also published three books about finite element applications for biomedical engineering and material modeling. This book, my fourth one, focuses on the material models associated with fracture and damage. Some errors and drawbacks most likely exist in the book due to the limitations of my knowledge. I welcome any feedback from the readers to improve the book.

It took me almost one year to complete this book. During the writing process, I received much support from my friends. I discussed the book's scope with Dr. J. S. Lin, Frank Marx, Dr. Adi Adumitroaie, and Dr. Jobie Gerken. I also consulted on some technical details with a long list of colleagues. Without their help, I would not have finished this book. In addition, Ronna Edelstein has again helped me with my English writing, which I greatly appreciate. I am grateful to the CRC Press staff, including Allison Shatkin and Ariel Finkle, for publishing my work. The book reflects my understanding of both material science and finite element modeling. I would like to express my sincere gratitude to Professor Si-wei Zhang and Dr. Patrick Smolinski for their invaluable teaching, guidance, and support during my time in graduate school. Finally, I thank my family for their constant strong support.

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# 1 Introduction

In traditional mechanical design, the material's ultimate strength was used for the failure criteria. Later, people found that the material failed, even though the material stress was much less than the material ultimate strength, which was due to the existence of the cracks. This marked the beginning of modern fracture mechanics that studies the crack initiation and growth. In addition to fracture mechanics, L. M. Kachanov and Y. N. Rabotnov proposed damage mechanics to study the initiation, propagation, and fracture of materials in phenomena. Unlike fracture mechanics, damage mechanics studies the material's properties prior to the existence of the macrocracks. Both fracture mechanics and damage mechanics describe the material damage process. They are playing more and more important roles in modern mechanical design; when linked with the stress and deformation of the structure, this design requires numerical simulation, such as finite element analysis, because of complex geometry and loadings. Therefore, this book focuses on the application of finite element analysis for fracture and damage mechanics.

As computer technology develops, the finite element method has been used widely in modern mechanical design. Over the past fifty years, hundreds of commercial software have been developed, in which ANSYS MAPDL occupies the leading position of structural analysis. Therefore, all examples in this book are simulated in ANSYS, and all input files are attached in the appendixes.

The book is organized into three parts. The first part focuses on background like the elasticity theory, finite element method, and various material models. After Chapter 1 introduces the subject, Chapter 2 presents the finite element method. A brief introduction of the finite element method at the beginning is followed by a list of the typical finite element procedure and a widely used language to build the finite element model – Ansys Parametric Design Language (APDL). Two examples of APDL are given at the end of Chapter 2, including one uniaxial tension test and another to simulate a plate with a hole under tension.

Chapter 3 discusses various material models. Section 3.1 covers linear isotropic elasticity, linear orthotropic elasticity, and their definitions in ANSYS. Section 3.2 focuses on some common elastoplastic material models such as isotropic hardening and kinematic hardening. Section 3.3 presents the creep material model, including the three creep stages, two creep types like explicit and implicit, and the thirteen creep equations in ANSYS. Section 3.4 briefly introduces the concept of material damage and damage types.

Part I introduces the various material models, and Part II turns to the fracture mechanics relevant to these material models. Chapter 4 initially gives the definition of the stress intensity factor, then computes the stress intensity factor of the central crack compared to ANSYS results using two different approaches. After that, it

simulates the 2D inclined crack problem and 3D single edge crack problem. The end of Chapter 4 discusses the crack growth with one example of the single edge crack problem.

Chapter 4 examines the stress intensity factor associated with linear elastic materials. When the materials are modeled as plasticity, we must select  $J$ -Integral, which is the topic of Chapter 5. Chapter 5 introduces the definition of  $J$ -Integral. It then calculates  $J$ -Integral of the central crack problem and compares the results against those of ANSYS. After a simulation of 2D and 3D penny shaped crack problems, Chapter 5 studies crack growth of a 3D penny shaped crack problem.

Different from Chapters 4 and 5, Chapter 6 focuses on a fracture parameter dependent on time, which is the  $C^*$ -Integral that links with the creep material model. After a discussion of definition and calculation of the  $C^*$ -Integral, Chapter 6 simulates the  $C^*$ -Integral of the standard compact type specimen.

The stress intensity factor,  $J$ -Integral, and  $C^*$ -Integral work with isotropic materials. The Virtual Crack Closure Technique (VCCT) in Chapter 7 works with composites. Chapter 7 concludes with a focus on the definition and calculation of VCCT, a simulation of 2D and 3D ELS problems using VCCT, and a study of crack growth of the double cantilever beam problem.

Part III discusses material damage. Chapter 8 focuses on the ductile damage model. Once ductile damage initiation and ductile damage evolution models are introduced, a uniaxial tension is analyzed using the ductile damage model and compared against ANSYS results. After that, the damage evolution of a notched model is simulated at the end of Chapter 8.

Creep damage is the topic of Chapter 9. A usermat subroutine is developed to implement creep damage using the Newton-Raphson method and validated by Python code. Next, the creep damage of the notched model is studied using the developed subroutine. At the end of Chapter 9, the material failure is implemented using a userfld subroutine.

Chapter 10 presents the fiber-reinforced material damage model, covering the definition of damage initiation and damage evolution. It also includes a study of the uniaxial tension test and provides a simulation of the damage evolution of a composite plate under tension.

The last chapter is a retrospective one. Based on the preceding three parts, Chapter 11 discusses the various material models, such as elastic models, elastoplastic models, and creep models; their corresponding fracture parameters, like stress intensity factor,  $J$ -Integral and  $C^*$ -Integral; and their corresponding damage models, which include the ductile damage model, creep damage model, and fiber-reinforced material damage model.

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